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Quantum radiation reaction in laser-electron beam collisions

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It is possible using current high intensity laser facilities to reach the quantum radiation reaction regime for energetic electrons. An experiment using a wakefield accelerator to drive GeV electrons into a counterpropagating laser pulse would demonstrate the increase in the yield of high energy photons caused by the stochastic nature of quantum synchrotron emission: we show that a beam of 10^9 1 GeV electrons colliding with a 30 fs laser pulse of intensity $10^{22}\,\mathrm{Wcm}^{-2}$ will emit 6300 photons with energy greater than 700 MeV, $60\times$ the number predicted by classical theory.

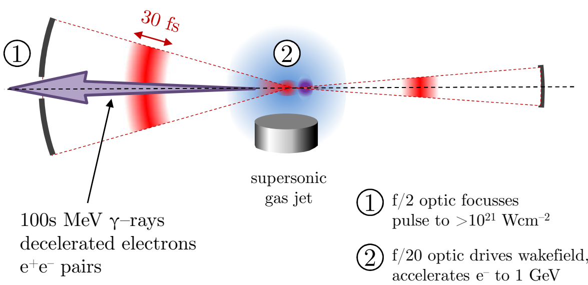

FIG. 1. (Color online). Diagram of an experimental geometry that could demonstrate quantum radiation reaction and pair production. The GeV electrons decelerate in large fields at the laser focus, producing gamma rays that pass through a hole in the f/2 optic.

Now that the power focussed in laser facilities exceeds 1 PW, electron dynamics enters a regime between classical and QED physics. In this letter we will consider an experimental setup where this transition can be explored: the collision of a GeV electron beam and a high-intensity laser pulse operating close to the current intensity frontier ($10^{22}\,\mathrm{Wcm}^{-2}$), as shown in Fig. 1. Electron dynamics in this regime is dominated by radiation reaction [1, 2], which will manifest itself in the efficient conversion of electron energy to hard gamma rays. We show that with current high-intensity laser facilities, it is possible to provide the first demonstration of the probabilistic nature of radiation reaction in the multi-photon, strong-field QED regime. The stochastic nature of this process increases its efficiency by over an order of magnitude compared to classical radiation reaction, and provides the most sensitive diagnostic of quantum synchrotron emission.

This experimental setup will provide a means of testing the fundamental physics underlying more exotic phenomena, such as pair cascades [3, 4], which occur in pulsar magnetospheres [5, 6] and are predicted to become significant above a threshold between 10^{23} and $10^{24}\,\mathrm{Wcm}^{-2}$. In these cases the plasma dynamics is strongly affected by the generation of macroscopic electron-positron pair plasmas, for example at the laser focus in laser-solid target interactions [7].

We parameterise the importance of strong-field QED effects with the quantity $\eta = |F_{\mu\nu}p^\nu|/mcE_{\mathrm{Sch}} \simeq \gamma|\mathbf{E}_\perp + \mathbf{v}\times\mathbf{B}|/E_{\mathrm{Sch}}$ [8], for an electron with four-momentum $p^\mu = \gamma m(c,\mathbf{v})$ in an electromagnetic field $F_{\mu\nu}$ with magnetic component $\mathbf{B}$ and component of the electric field perpendicular to the electron direction of propagation $\mathbf{E}_\perp$. η is the ratio of the electric field in the electron rest frame to $E_{\mathrm{Sch}} = m^2c^3/e\hbar$, the characteristic field of QED [9]. In the setup we consider, the probabilistic nature of photon emission is evident for $\eta \sim 0.1$. The QED-dominated regime is reached when $\eta > 1$; at $\eta = 90$, a 50 GeV electron would be capable of producing multiple pairs on passage through a petawatt laser pulse [10].

In a classical description, the electron travels on its worldline radiating continuously as it accelerates in the laser fields. The typical energy of a single emitted photon is $0.44\eta\gamma mc^2$ [8]; therefore as η approaches 1, a single photon becomes capable of carrying off a significant fraction of the electron's energy and the recoil of that emission must be taken into account. However, the quantum description of radiation reaction differs from classical theory in two ways.

Firstly, quantum corrections to the radiated spectrum cut off the tail of photons with energies greater than that of the electron [11] and include spin-flip transitions, the latter markedly increasing the probability of radiating photons with energies comparable to that of the electron [12]. These modifications means that the total radiated power is smaller than the equivalent classical power by a factor $g(\eta) \in (0,1)$ [13, 14].

The second and more significant effect for us is that the process of photon emission is stochastic [4, 15]; thus the electron has only a probability to emit a gamma-ray photon of given energy. This gives rise to a phenomenon called 'straggling' [15, 16], where the electron may propagate a significant distance through the strong laser fields without radiating. Since the laser pulse will have a spatial intensity profile, it is possible for the electron to reach the region of highest intensity at the centre having lost much less energy than a classical electron. The η of an electron that has straggled in this way will be boosted above that which could be reached classically. As the tail of the photon spectrum increases non-linearly with η, straggling enhances the yield of hard gamma-rays. Furthermore, these energetic photons can decay in the strong electromagnetic fields to produce electron-positron pairs by the Breit-Wheeler process [8]. Pairs can also be produced directly by the electron in the trident process.

These strong-field QED effects have been investigated experimentally with 100 GeV electrons incident on crystals at the CERN Super Proton Synchrotron [17] and in the collision of a 50 GeV electron beam and a $10^{18}\,\mathrm{Wcm}^{-2}$ laser pulse at the SLAC facility [18]. In the former, the reduction in radiated power $g(\eta)$ was measured and found to agree well with theoretical predictions; in the latter, pair production by gamma rays created by inverse Compton scattering was observed. However, by using a more intense laser, we can reach the same η with electrons of much lower energy. As it has been shown that PW lasers in long focus can generate GeV electron beams by wakefield acceleration [19–21], it is realistic to consider the experiment shown in Fig. 1, where another laser pulse, in tight focus, provides the high-intensity target for the wakefield-accelerated electron beam.

We have developed a Monte-Carlo algorithm to simulate the interaction of an intense laser pulse with an energetic electron beam, following §2 of [15] and §3 of [22]. The three processes included are: photon and (trident) pair production by an electron, and (Breit-Wheeler) pair production by a photon.

The rates of these three processes are calculated in the Furry picture of QED [23], where the electron interacts with both an external, unquantised electromagnetic field and a fluctuating component of the same [24, 25]. In between interactions with the latter (i.e. photon emission and absorption) it propagates classically [26]. The photon formation length is smaller than the laser wavelength by a factor of the laser strength parameter a_0 [27]; since $a_0 = [I_{\mathrm{L}}(\lambda/\mu\mathrm{m})^2/1.37\times10^{18}\,\mathrm{Wcm}^{-2}]^{1/2} \gg 1$, where I_{L} and λ are the laser intensity and wavelength respectively, we treat the emission process as pointlike and instantaneous. As the fields are quasi-static over the emission process, these rates may be calculated in an equivalent system of fields with the same instantaneous value of η, such as a static magnetic field (in the limit $B \to 0$, $\gamma \to \infty$) or a plane EM wave (in the limit of zero frequency) [27].

Using the static magnetic field approach, the spectrum of emitted photons is determined by the quantum synchrotron function $F(\eta, \chi)$, where $\chi = (\hbar/2mc^2)|\omega\mathbf{E}_\perp + c^2\mathbf{k}\times\mathbf{B}|/E_{\mathrm{Sch}}$ is the counterpart of the electron parameter η for a photon of frequency ω and wavevector $\mathbf{k}$. The rate of emission is given in Erber [13] and Baier *et al.* [14]. In the plane wave approach, this same process is called non-linear inverse Compton scattering [26, 28, 29].

We further assume that the electrons may be treated independently as they are highly energetic, with $\gamma \gg a_0 \gg 1$. The rigid-beam approximation is valid as the transverse momentum gained from the laser is small, and in the frame co-moving with a nearly mono-energetic beam, the energy of interaction between particles will also be small. Therefore the electron motion, initially antiparallel to the optical axis, is one-dimensional and constant between discrete photon emission events.

This algorithm is implemented thus: the code initially assigns to each electron a pseudorandom 'final' optical depth against photon emission τ_{ph} and trident pair production τ_{tri}. As they propagate through the laser pulse, it integrates their differential optical depths against trident pair production $\mathrm{d}\tau_{\mathrm{tri}}/\mathrm{d}t$ and photon emission $\mathrm{d}\tau_{\mathrm{ph}}/\mathrm{d}t = (\sqrt{3}\alpha/2\pi\tau_{\mathrm{C}})\eta h(\eta)/\gamma$, where α is the fine structure constant, τ_{C} the Compton time and $h(\eta) = \int_0^{\eta/2}\mathrm{d}\chi F(\eta,\chi)/\chi$ [13, 14].

Emission occurs when the electron reaches that 'final' optical depth [15]. Following that, a new final depth is assigned. The photon energy is found by pseudorandomly sampling the quantum synchrotron distribution [3, 10] $(\int_0^{\chi}\mathrm{d}\chi F(\eta,\chi)/\chi)/h(\eta)$. The electron energy, constant between emissions, is then reduced by the energy of the emitted photon. In the static magnetic field approach, this follows from energy conservation. Since synchrotron radiation does not conserve the component of momentum normal to the magnetic field [30], when implemented in the code, this leads to a fractional error $\Delta\gamma/\gamma \propto 1/\gamma_{\mathrm{emit}}$ which is negligible for all emission events [4, 7].

Energy loss to trident pair production is neglected, as the rate is calculated in the Weizsäcker-Williams approximation, which treats the exchanged virtual photon as real (i.e. the momentum transfer from electron to pair $q^2 \to 0$). Breit-Wheeler pair production is modelled by assigning to each photon at its creation a final optical depth τ_{BW}. The rate of pair production [13] is integrated along the photon trajectory to determine whether it decays to a pair.

We will compare our fully stochastic model of radiation reaction with a semi-classical model, which we call 'continuous radiation reaction'. In this model the electron loses energy continuously according to a damping term given by the Landau-Lifshitz force [30] modified to include the quantum correction to the radiated power: $\mathrm{d}\gamma/\mathrm{d}t = (2\alpha/3)(\eta^2/\tau_{\mathrm{C}})g(\eta)$. For consistency, photon spectra are obtained by sampling at each timestep the quantum synchrotron distribution. Thomas *et al.* [31] use a similar model to simulate laser-electron collisions for electrons with $\gamma_0 = 400$, where η is small enough that probabilistic effects can be neglected.

The parameters of the simulation are as follows: the laser pulse has wavelength $\lambda = 1\,\mu\mathrm{m}$, is linearly polarised and has Gaussian temporal profile with a full width at half-maximum (FWHM) of 30 fs. The electrons have ini-

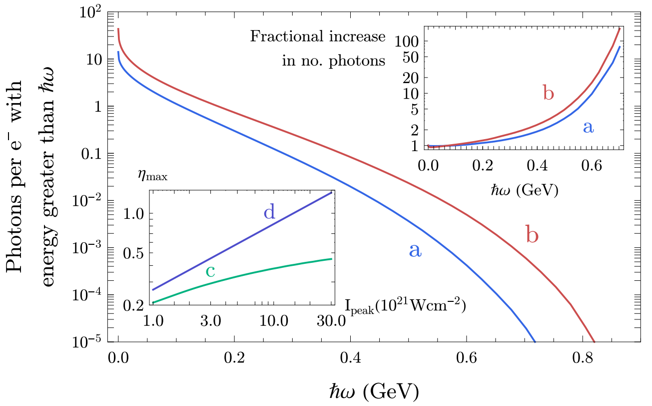

FIG. 2. (Color online). The number of photons with energy $> \hbar\omega$ for a 1 GeV electron incident on a linearly polarised, plane-wave laser pulse with Gaussian temporal profile (FWHM 30 fs) and peak intensity (a) 10^{21} (b) $10^{22}\,\mathrm{Wcm}^{-2}$. *Inset* (*top*) The fractional increase, due to straggling, in the number of photons with given minimum energy. *Inset* (*bottom*) The maximum η experienced by a (c) continuously and (d) discontinuously radiating GeV electron incident on the same pulse with given intensity. The increase in going from (c) to (d) is responsible for the hardening of the photon spectrum.

tial gamma factor $\gamma_0 = 2 \times 10^3$ and propagate along the optical axis antiparallel to the laser pulse; this ensures that the $\mathbf{E}_\perp$ and $\mathbf{v} \times \mathbf{B}$ terms in the definition of η are additive, i.e. $\eta \approx 2\gamma E_\mathrm{L}/E_\mathrm{Sch}$. Each simulation follows at least 10^7 macroelectrons, their trajectories discretised into intervals of $\Delta t = 10^{-17}$s. The laser fields are calculated at each timestep using the classical solution for a Gaussian focussed beam.

If the laser pulse is a plane wave, the number of photons per electron with energy greater than $\hbar\omega$, and the increase in the same due to straggling, are given in Fig. 2. The boost to the electron's maximum η as a result of straggling is also shown: this boost is responsible for the hardening of the photon spectrum.

In reality both the laser and electron beam will have temporal and spatial structure. As long as enough of the beam is incident on the laser focus, there will still be strong evidence of probabilistic photon emission. Consider a beam of 1 GeV electrons propagating antiparallel to the laser but uniformly distributed around the optical axis in a disk of radius 10 μm. The laser pulse, still linearly polarised, is focussed with Gaussian temporal and radial profiles (FWHM 30 fs and waist size 2.2 μm). If it has peak intensity of $10^{22}\,\mathrm{Wcm}^{-2}$, the number of photons per electron with energy greater than 500 (700) MeV this interaction produces will be 7.40×10^{-4} (1.32×10^{-5}), $4.3\times$ $(160\times)$ greater than that which would be radiated classically.

It is then possible to distinguish discontinuous radiation reaction by measuring the high-energy tail of the gamma-ray spectrum, as the yield of these photons is

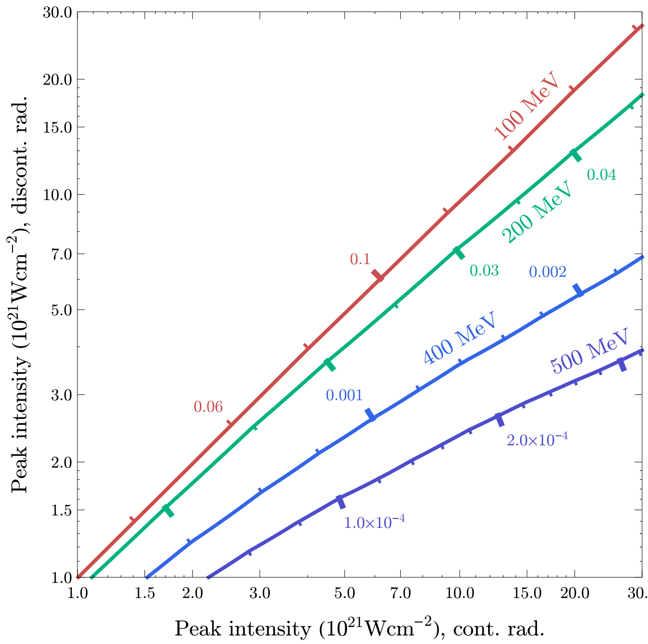

FIG. 3. (Color online). Indicated along the coloured lines are the numbers of photons per electron with energy greater than 100 MeV (red), 200 MeV (green), 400 MeV (blue) and 500 MeV (purple) if the electrons radiate discontinuously (vertical axis) or continuously (horizontal axis). The interaction here is between a focussed laser pulse of given peak intensity (FWHM 30 fs, waist 2.2 μm) and a beam of 1 GeV electrons (radius 10 μm around the optical axis).

most enhanced by straggling. Fig. 3 shows that if the electrons radiate semi-classically, rather than stochastically, the laser pulse must be much more intense for the interaction to produce the same number of high-energy photons. This increase in intensity is necessary because a stochastically radiating electrons can reach a higher η.

Assessing the nature of the emitted radiation in a laser-electron beam experiment can then done by fitting the observed spectrum to the points in Fig. 3. For example, the detection of 9.0×10^{-2} photons per electron with energy greater than 100 MeV would be consistent with either a continuously or discontinuously radiating beam colliding with a laser pulse of peak intensity $5 \times 10^{21}\,\mathrm{Wcm}^{-2}$. In contrast, the simultaneous detection of 1.9×10^{-3} photons per electron with energy greater than 400 MeV would only be consistent with discontinuous radiation reaction, as a continuously radiating beam would have to collide with a pulse of peak intensity $1.8 \times 10^{22}\,\mathrm{Wcm}^{-2}$ that number of gamma rays. If the number of photons with energy greater than 500 MeV is measured instead, that equivalent intensity rises to $5.2 \times 10^{22}\,\mathrm{Wcm}^{-2}$.

A typical laser wakefield will accelerate a bunch of 10^9 electrons, which will produce a beam of 4.1×10^5 photons with $\hbar\omega > 500$ MeV, collimated within an angle of 30

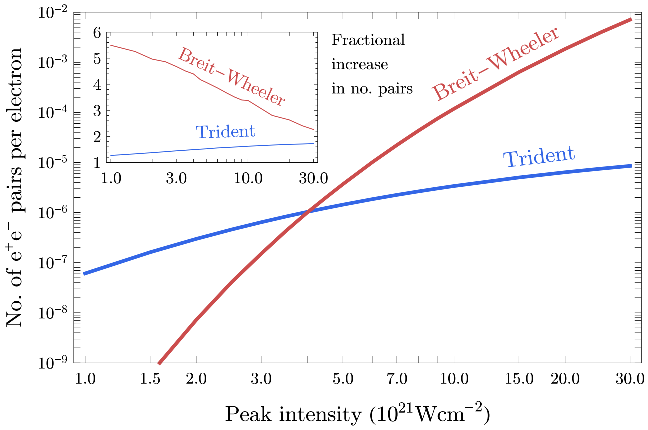

FIG. 4. (Color online). The number of Breit-Wheeler (red) and trident (blue) electron-positron pairs produced per 1 GeV electron colliding with a linearly polarised plane-wave laser pulse with Gaussian temporal profile (FWHM 30 fs) and given intensity. *Inset* The fractional increase in the number of pairs for the same processes produced by treating radiation reaction probabilistically.

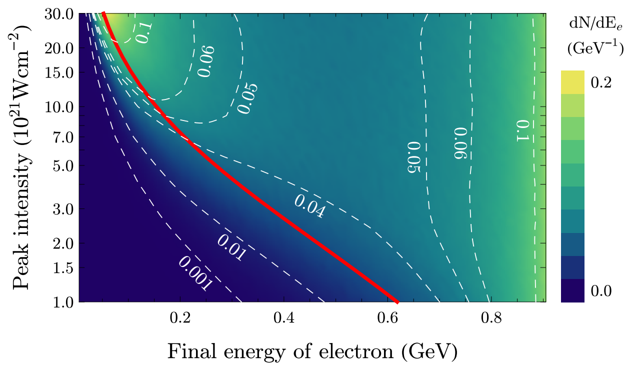

FIG. 5. (Color online). The final state energy distribution dN/dE_e of a 10 μm radius beam of 1 GeV electrons incident on a pulse with given peak intensity. White, dashed lines are contours of constant dN/dE_e. The red line is the lowest energy that can be reached with classical radiation.

mrad. When shielded to block lower energy gamma rays, this will be sufficient energy deposition in a calorimeter to provide a good signal of discontinuous radiation reaction.

Our code also models pair production by both the gamma rays and the electrons. Shown in Fig. 4 are the number of pairs produced per electron by a monoenergetic 1 GeV beam incident on a plane-wave laser pulse with given peak intensity.

Pair production will form a key part of the new physics probed by future high-intensity laser experiments. However, pairs do not become energetically significant in the experimental setup we describe here because they are not accelerated by the laser fields; thus their probability of emitting a photon or creating an additional pair before exiting the laser pulse is very small. Converting laser energy to pairs requires much more intense, counterpropagating laser beams that can accelerate the electron up to high energy between emission events [4, 15, 27].

The large increase in the number of Breit-Wheeler pairs shown in the main part of Fig. 4 arises because the rate $d\tau_{\rm BW}/dt$ is highly non-linear in χ and has maximum rate of growth for $\chi = 0.62$. Reaching this switch-on with current high-intensity lasers is strongly dependent on the yield of close to GeV photons. As straggling enhances this yield, it also increases the yield of pairs (see inset of Fig. 4). The number of trident pairs is similarly increased, as straggling allows more high-energy electrons to penetrate to the region of highest field intensity.

Although potentially more difficult to measure, the electron energy distribution can provide evidence of quantum radiation reaction [32]. In Fig. 5, we show how the initally monoenergetic beam acquires a spread because the electrons counterpropagate with a range of displacements from the optical axis, experiencing different laser intensities. The energy loss of electrons subject to continuous radiation reaction is determined only by laser intensity. Thus there is a non-zero lower bound to the classical final state energy distribution, corresponding to those electrons which have passed through the region of highest laser intensity.

Discontinuously radiating electrons lose energy probabilistically, so some electrons will straggle and lose more energy than possible classically. At 10^{22} Wcm^{-2}, 6.1×10^{-3} of the beam electrons will experience an energy loss exceeding 863 MeV, the maximum possible classically. It may be possible to distinguish these electrons if the initial energy of the beam is very well-characterised and devoid of a low energy tail. This could be accomplished with a particle accelerator [18], or with magnetic filtering of the electron beam produced by a laser-driven wakefield.

In this letter we have considered the effects of including a fully stochastic model of radiation reaction on the motion of an energetic electron beam incident on an intense laser pulse. Under the chosen conditions, the parameter $\eta = 2\gamma E_L/E_{\rm Sch} \sim 0.1$ and so QED effects must be included. We find that electron motion is dominated by quantum radiation reaction, enhancing the yield of photons with $\hbar\omega > 700$ MeV by a factor of 160; thus the collision of a GeV electron beam with a petawatt laser can provide evidence of both stochastic gamma ray and pair production. The observation of 700 MeV photons will be an unambiguous signature that GeV electrons have been incident on, and straggled through, the region of highest intensity at the laser focus. The measurement of increased energy spreading of the electron beam is likely to be more ambiguous in a realistic experiment where laser parameters vary from shot to shot. Experimental validation of the probabilistic nature of QED-plasma models will underpin the simulation and design of the next generation of higher energy laser-plasma interactions.

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